CSE 539: Applied Cryptography
RSA + DHKE + PK

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Reading: https://joyofcryptography.com/pdf/chap15.pdf

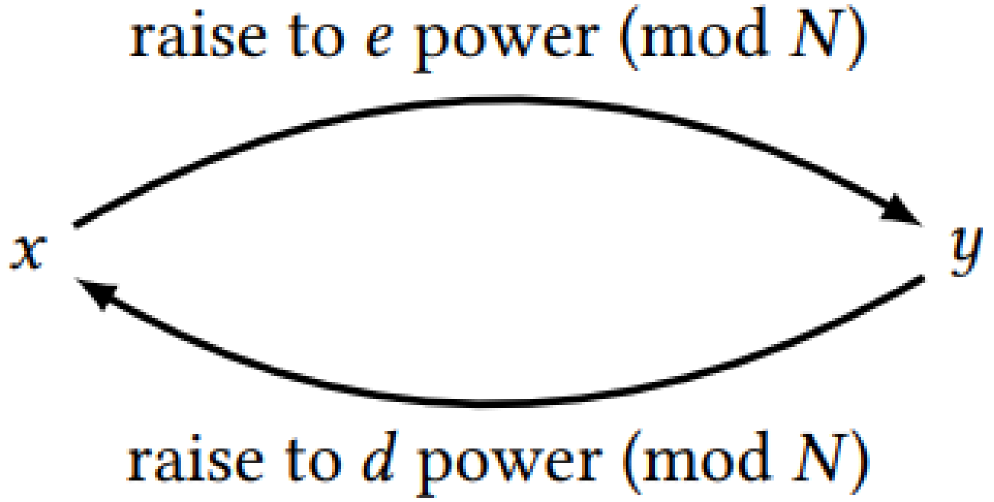
RSA

The RSA function is defined as follows:

- Let p and q be distinct primes (later we will say more about how they are chosen), and let $N = pq$. N is called the **RSA modulus**.
- Let e and d be integers such that $ed \equiv_{\phi(N)} 1$. That is, e and d are multiplicative inverses mod $\phi(N)$ — not mod N!
- The RSA function is: $x \mapsto x^e \,\%\, N$, where $x \in \mathbb{Z}_N$.
- The inverse RSA function is: $y \mapsto y^d \,\%\, N$, where $x \in \mathbb{Z}_N$.

RSA Security

- Given only the public information (N, e), it should be hard to compute the RSA inverse (y -> y^d mod N) on randomly chosen values.
  - In other words, the only person who is able to compute the RSA inverse function is the person who generated the RSA parameters



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- For example, given $\delta = |p^2 - q|$

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- For example, given that p and q are close (e.g. $|p - q| < 1000$)

DHKE

Alice		Bob
$a \leftarrow \mathbb{Z}_n$	$A = g^a \ \%p \longrightarrow$	
	$\longleftarrow B = g^b \ \%p$	$b \leftarrow \mathbb{Z}_n$
return $B^a \ \%p$		return $A^b \%p$

Definition 14.2 (Discrete Log) *The* ***discrete logarithm problem*** *is: given $X \in \langle g \rangle$, determine a number x such that $g^x = X$. Here the exponentiation is with respect to the multiplication operation in $\mathbb{G} = \langle g \rangle$.*

Diffie-Hellman Key Agreement

- Quiz Sample:

Consider the following key-exchange protocol where p, g, q are public parameters

(i) Alice chooses a random exponent $a_1 \leftarrow \mathbb{Z}_q$ and computes $h_1 = g^{a_1} \bmod p$. Alice sends h_1 to Bob

(ii) Bob chooses two random exponents a_2, a_3, and computes $h_2 = g^{a_2+a_3} \bmod p$. Bob sends h_2 to Alice.

(iii) Alice outputs a shared key $k = h_2^{a_1} \bmod p$

Show how Bob outputs the same key k?

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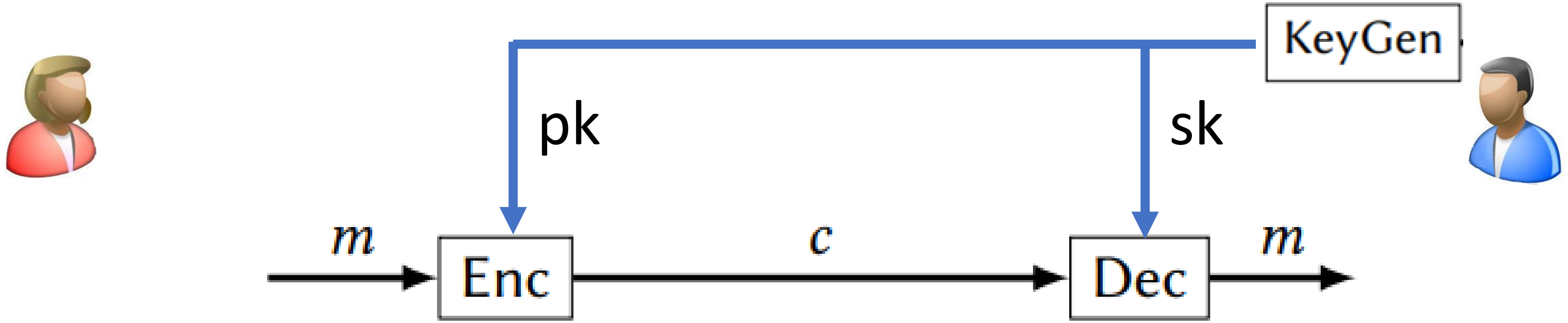
Public Key



Public Key: ElGamal Encryption

ElGamal encryption is a public-key encryption scheme that is based on DHKA. Given a choice of cyclic group $\mathbb{G}$ with n elements and generator g, the construction of ElGamal encryption is as below:

Keygen:	Enc($A, M \in \mathbb{G}$):	Dec($a, (B, X)$):
$sk := a \leftarrow \mathbb{Z}_n$	$b \leftarrow \mathbb{Z}_n$	return $X(B^a)^{-1}$
$pk := A := g^a$	$B := g^b$	
return (sk, pk)	return $(B, M \cdot A^b)$	

ElGamal Encryption

Suppose you do not know the secret key sk. Given the public key pk and the ElGamal ciphertext (B, X) that encrypts an unknown plaintext $M \in \mathbb{G}$, construct another ElGamal ciphertext (B', X') that decrypts to the same M (e.g., show how to do it without knowing M). Show the correctness of your construction.

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